Downloaded from rsta.royalsocietypublishing.org

Transition to turbulent flow in aerodynamics

Robert I. Bowles

Phil. Trans. R. Soc. Lond. A 2000 **358**, 245-260
doi: 10.1098/rsta.2000.0530

Email alerting service

Receive free email alerts when new articles cite this article - sign up in the box at the top right-hand corner of the article or click **here**

To subscribe to *Phil. Trans. R. Soc. Lond. A* go to:
http://rsta.royalsocietypublishing.org/subscriptions

This journal is © 2000 The Royal Society

Downloaded from rsta.royalsocietypublishing.org

Transition to turbulent flow in aerodynamics

By Robert I. Bowles

Department of Mathematics, University College London, Gower Street, London WC1E 6BT, UK

The difficult problem of understanding the physical mechanisms at work in the change from laminar, or smooth, flow to random turbulent flow, with its wide range of active time- and space-scales, has occupied engineers, physicists and mathematicians for the past century. When an airfoil is placed in and parallel to a current of fast-moving air, a so-called boundary layer forms on its surface as the velocity of the air at the surface must be reduced to zero. Near the front of the airfoil the boundary-layer flow is smooth and steady, but further downstream it is seen to become highly irregular, unsteady and turbulent, often at a well-defined front. Scientists working in transition prediction aim to answer the questions of where and why this transition occurs. It is a problem distinct from, but related to, that of understanding turbulent flow itself. It is of prime industrial importance. For example, a turbulent flow offers more drag resistance, and, indeed, an aircraft designed so that more of the flow over its wings is laminar can carry more passengers and much less fuel. Understanding the physical structures and flow patterns visible in the late end stage of transition and the initiation of turbulent spots, isolated patches of turbulence surrounded by laminar flow, should also throw much-needed light on the structures seen in fully developed wall turbulence and help in the equally difficult, distinct, problem of modelling turbulent flow.

There are many possible routes through transition, depending on the flow configuration and geometry and the method in which transition is initiated by any of the range of possible background disturbances present, either in the free stream or in the form of roughness on the surface, for example. In the past 20 years, techniques for tracing the linear and nearly linear growth of small disturbances in the boundary layer have been developed that could form part of effective design tools for engineers. There has also been an increased theoretical understanding, made possible by the application of high Reynolds number asymptotic theories, of the myriad of possible interactions between disturbances driving this relatively slow stage of the transition process. Much important work remains to be done to include in any design tool the important processes occurring at the two ends of this process. Firstly, how do disturbances enter the boundary layer to be amplified, known as the receptivity problem, and secondly, what happens at the end stage, where the disturbances have grown so large that the nearly linear theories are no longer applicable?

Recent experimental work has shown a remarkable similarity in the characteristics of this final breakdown among a variety of flows. Two-dimensional flows, such as that over a plate aligned with the flow or in a channel or pipe, gradually develop three-dimensional structures, known as lambda vortices. These then rapidly break down in two distinct ways, which are both active almost simultaneously. One gives rise to spikes: short-lived, large-amplitude pulses, which are practically deterministic

© 2000 The Royal Society

Downloaded from rsta.royalsocietypublishing.org

in nature. The second involves a secondary instability and the initiation of random fluctuations. Three-dimensional flows, such as those on swept wings, develop cross-flow vortices, which themselves seem to break down via a secondary instability mechanism, possibly similar to that seen in lambda vortices.

This article reviews recent developments in the field of transition research, concentrating on those related to the late stages of breakdown and the onset of random behaviour. It brings together results from young experimentalists, computationalists and theoreticians and looks forward to an increased understanding of this challenging and important problem.

Keywords: boundary-layer transition; spikes; lambda vortices

1. Introduction

The prediction of the point of transition from smooth to turbulent flow is a problem of immense importance to industry. Turbulent flow is characterized by a wide range in the scales of regions of circulating flow known as eddies. These circulations are able to transport heat, momentum and tracers such as chemical reactants more effectively than is the case in laminar flow. This has many implications. We may want to provoke transition to ensure mixing of air and fuel in combustion. A turbulent flow over the latter half of a wing ensures a transport of high momentum to fluid close to the airfoil. This increases its inertia and helps prevent separation of the flow from the wing, which would lead to a dramatic drop in lift with disastrous consequences, particularly during landing. In flight conditions, in contrast, the same physical effect leads to an increase in the drag on the wing and turbulent flow is, therefore, to be avoided here. It has been calculated that fuel savings of 20% could be possible for an airliner designed so that much of the flow over it is laminar. In jet engines, the exhaust gases are at very high temperatures, and an increased heat transfer in any turbulent flow over the rotor blades leads to their rapid heating and a reduction in their lifespan. A similar problem arises at the very high (hypersonic) speeds of the reentry of reusable launch vehicles such as the space shuttle, where increased heat transfer can lead to degradation of protective tiles.

Away from the field of aeronautics, the transition phenomenon is a limiting feature for computer codes, which aim to predict the fluid flow in many applications, engine and pump design for example. They do so by solving the governing equations—the Navier–Stokes (NS) equations—numerically. However, transition is associated with a rapid cascade of motions towards shorter time- and space-scales and the codes have to resolve these new features accurately together with the larger-scale original motions. Computers with such speed and storage capabilities are still many years away. Direct numerical simulation (DNS) of the transition process is presently only possible in simplified geometries. In engineering applications, this problem is often approached through the use of empirical models for the prediction of the transition point. A separate turbulence model can then be used downstream of this point. However, the process of transition is a complex phenomenon affected by many inputs, and a model successful in one situation may fail dismally in another.

There are many different routes to transition. Probably the best understood is the so-called K-type transition of the planar flow over a flat plate (Klebanoff *et al.* 1962). We will concentrate on this flow, primarily, because the features seen in the

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Downloaded from rsta.royalsocietypublishing.org

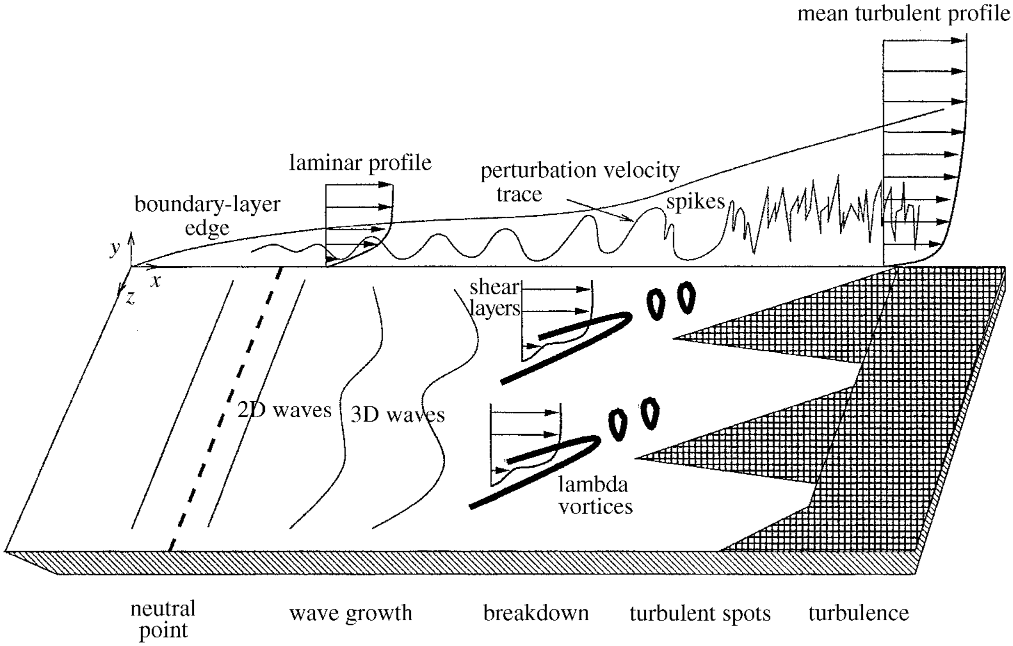


Figure 1. An overview of the transition process.

late stages of transition appear to be in common with those seen in more complicated flows, which may exhibit different phenomena in the earlier stages. If the airflow in a wind tunnel has a sufficiently low turbulence level, the flow over a flat plate aligned with the flow may be disturbed with an artificial, controllable, planar forcing and the disturbance traced downstream (see figure 1). In flight conditions, the transition process is so rapid that it takes place at a well-defined front, as mentioned in the abstract. If the initial amplitude is sufficiently small, so-called Tollmien–Schlichting (TS) waves are seen. These instability waves are named after the German workers of the 1920s and 1930s, who predicted their existence theoretically (Heisenberg 1924; Tollmien 1929; Schlichting 1933), although they were not observed in experiments until wind tunnels had become of sufficiently high quality (Schubauer & Skramstad 1943). Incidentally, at altitude, the disturbance level is usually much less than that attainable in a wind tunnel. Recent work aimed at developing efficient methods for predicting the growth rate of these waves is described in § 2 of this paper. A promising practical approach, not described here, is to solve the so-called parabolized stability equations (PSEs) (Herbert 1997), which model the NS equations and attempt to describe a relatively small disturbance developing slowly in a growing boundary layer. This is a useful engineering tool but it is unable to describe the rapid breakdown to short scales described below.

The two-dimensional waves themselves then develop into growing three-dimensional structures by a so-called secondary instability process. These develop nonlinearly into lambda (Λ)-shaped structures, known as lambda vortices, and associated strong shear layers: regions in which the fluid velocity changes rapidly. These features are described further in § 3 below. When amplitudes reach sufficiently large values, there is a rapid breakdown to short-scaled structures known as spikes, named after the

Downloaded from rsta.royalsocietypublishing.org

spikes seen in traces of perturbation velocity against time. This is followed by the onset of random behaviour and the eventual development of a turbulent flow. This is often through the growth of isolated patches of turbulence, or spots, from the regions of the spikes that merge as they travel downstream. An excellent description of this process is given by Kachanov (1994). Recent experimental results, and new experimental techniques illustrating these latter stages, are described in § 3. Strong shear layers, Λ-shaped structures and their subsequent breakdown are also seen in so-called N-type transition of the flow over a flat plate, which occurs at lower input amplitudes (Bake *et al.* 2000), in pipe flows (Han *et al.* 2000), and in flow over compliant surfaces (Metcalfe *et al.* 1991). Strong shear layers, and their breakdown to random disturbances, are seen in the transition of boundary layers in which the flow is not in a single direction during the latter stages of so-called cross-flow instability (Wintergerste & Kleiser 1995; Lerche 1997).

Fully developed turbulent flow over a surface is a complicated, three-dimensional phenomenon. However, structures may be identified within the flow and many of these have similarities with the structures seen in the late stages of the transition process. From a theoretical point of view, the similarities are especially strong between the mechanism of the breakdown of lambda vortices and that of the eruption of fluid from regions close to the surface, which occurs in turbulent flow (Walker 1990*a*, *b*; Li *et al.* 1998). An understanding of these features gained from the study of transition could help in the study of how turbulence is maintained against the natural dissipation of the energy in the flow.

The NS equations are nonlinear and, at moderate speeds, have a single non-dimensional parameter, the Reynolds number Re. This is the ratio UL/ν, where U and L represent typical velocity and length-scales, respectively, for the flow, and ν is the kinematic viscosity, a measure of the 'stickiness' of the fluid. The Reynolds number for a wing in flight is typically 10^8. The inverse of the Reynolds number multiplies the terms in the NS equations representing viscous diffusion of momentum in the fluid. At large Re, one might therefore presume that viscous effects are unimportant. However, where the fluid is in contact with a solid body, such as a wing, the velocity of the fluid relative to the body must be zero. Viscosity acts in a thin layer—the boundary layer around the body—to reduce these velocities to zero. This is an example of the separation of the space- and time-scales, over which different physics is active, which occurs at large Re. The theoretical work described in § 4 takes advantages of this separation and derives reduced sets of equations successful in describing some features of breakdown.

2. The instability mechanism

From the theoreticians point of view, the problem of transition is viewed as a nonlinear stability problem. The solution for the steady boundary-layer flow over the flat plate—the so-called Blasius solution—describes how the boundary-layer thickness grows downstream from zero at the leading edge due to a balance of inertial and viscous effects, growing more slowly for higher Re. A first step then is to look for small-amplitude wave-like perturbations of a given frequency to this flow and so to neglect the nonlinear terms in the NS equations. If the boundary-layer growth is also neglected, this leads to the Orr–Sommerfeld (OS) equation governing the stability of the flow local to some point on the plate (Orr 1907; Sommerfeld 1908). This process

Downloaded from rsta.royalsocietypublishing.org

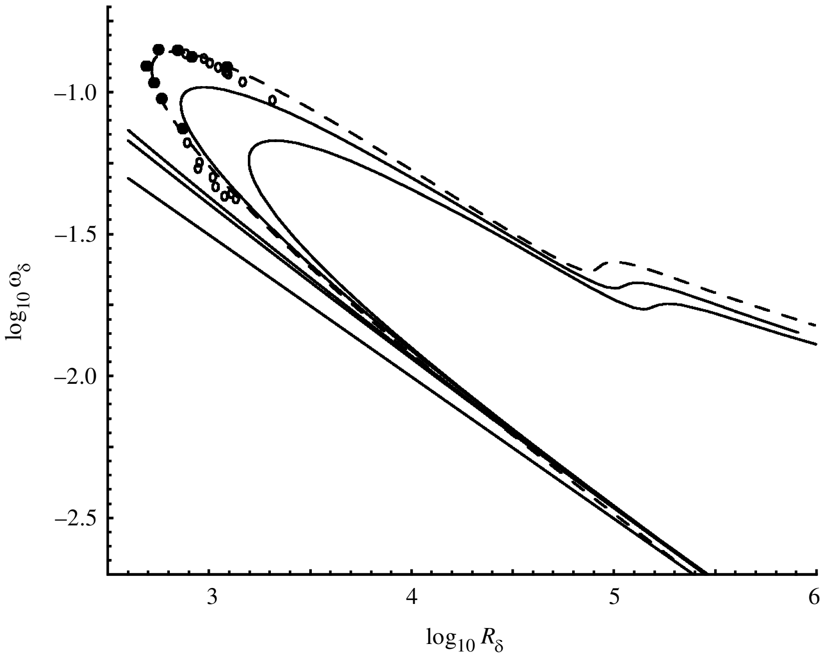


Figure 2. The neutral curve of the Blasius boundary layer calculated from OS equation (– – –) and experimental points from Ross *et al*. (1970) (∘) and Klingmann *et al*. (1993) (•). Also shown are asymptotic predictions for the lower branch for large R_δ and predictions from the composite expansions of Healey (1995), which straddle both branches (——).

neglects viscous effects in discarding the velocity profile's development, but retains viscous terms in the OS equation. Indeed, viscosity is essential in the growth mechanism for TS waves (Lighthill 1963). The predicted length-scale of the TS waves is shorter than the scale over which boundary-layer growth occurs, however, so this can be a good first approximation to reality.

A common choice for the Reynolds number R_δ appearing in the OS equation is based on the local boundary-layer thickness, δ, and the local streamwise velocity outside the boundary layer, U_∞. It grows downstream from zero at the leading edge as the boundary layer grows and scales as the square root of the global Reynolds number *Re*. Figure 2 shows the so-called neutral curve in the R_δ–ω_δ plane, with ω_δ a non-dimensionalized wave frequency. Disturbances lying on this curve are neutral, meaning that they have zero growth rate, while those inside grow so that the curve can be used to predict which disturbances grow at any point in the boundary layer. All disturbances decay for $R_\delta \lesssim 520$, the neutral point, explaining the delay in the onset of transition from the leading edge. The figure also includes experimental measurements of the neutral curve, illustrating the range of Reynolds numbers over which laminar flow may be maintained in a good wind tunnel (Ross *et al*. 1970; Klingman *et al*. 1993).

The lower and upper portions of the curve are known as the lower and upper branches, respectively. The structure of TS waves at large R_δ was elucidated by Reid (1965), Smith (1979*a*) and Bodonyi & Smith (1981), who showed that lower- and upper-branch neutral waves have frequencies that scale with $R_\delta^{-1/2}$ and $R_\delta^{-1/5}$,

Downloaded from rsta.royalsocietypublishing.org

respectively. The difference in scales is due to the slightly different physics active in maintaining the wave motion. Viscosity may be neglected except in a thin layer close to the plate and in the so-called critical layer located where the streamwise velocity of the Blasius flow is equal to the speed of the wave. Both these layers are relatively close to the plate, where the velocity profile may be approximated by a linear increase from zero at the plate, thus neglecting the curvature of the profile. For the lower frequencies close to the lower branch, these two layers merge into a single region and can interact. Viscosity is then able to destabilize the flow. As the frequency increases, the layers separate, interact less strongly and the growth rate of the waves is reduced. Finally, close to the upper branch, the weak stabilizing effect of the curvature of the Blasius flow is felt and the waves become neutral and decay. Much theoretical work, concentrating on disturbances close to the upper branch, has been pursued by Goldstein (1995). The theory described in § 4 of this paper, in contrast, looks at disturbances with a lower-branch structure.

In fact these predictions for the neutral frequencies at high R_δ had been known for some years from a study of the OS equation (see, for example, Reid 1965). The contribution of the later authors lay in the casting of the predictions as 'rational' solutions of the NS equations. This has the technical meaning that the magnitude of each neglected term in the equations, and so of the physical effects they represent, is associated with a particular inverse power of R_δ, and can, in principle, be included in the prediction by taking more terms in an expansion as R_δ becomes large. Although accepted as being of immense theoretical value in clarifying the mechanism of the waves, the practical use of such approaches is sometimes limited as the parameter assumed small in the expansion is often not small at Reynolds numbers of practical interest. However, the expansion can be continued to include the growth of the boundary layers; the predictions for the lower branch are illustrated in figure 2. Jonathan Healey of the University of Keele has shown that the predictions for the upper branch only hold good to the right of the kink in the upper branch (Healey 1995). At transition Reynolds numbers, he has shown that TS waves have the character of lower-branch disturbances even along the upper branch, in that the viscous layers mentioned above remain merged. He has recently generalized this expansion approach, extending the work of Hultgren (1987), and used symbolic algebra packages to generate results valid at transition Reynolds numbers over both branches (see figure 2). These predictions include the effects of boundary-layer growth and it is hoped that this work could lead to an alternative technique for the calculation of disturbance growth rates as opposed to the use of the Orr–Sommerfeld equation that, we recall, neglects boundary-layer growth and is significantly faster than the alternative PSE approach. Furthermore, the method may be extended to include a finite disturbance amplitude by incorporating the nonlinear terms in the NS equations at transition R_δ, so extending the predictions of Smith (1979*b*), which are valid for $R_\delta \gg 1$.

3. Breakdown

The process of the nonlinear development and breakdown of the TS waves has been clarified by recent careful experiment and by DNS of the transition process. Interesting three-dimensional structures are seen to develop. The condition of zero velocity at the plate surface causes the velocity necessarily to increase from zero towards the

Downloaded from rsta.royalsocietypublishing.org

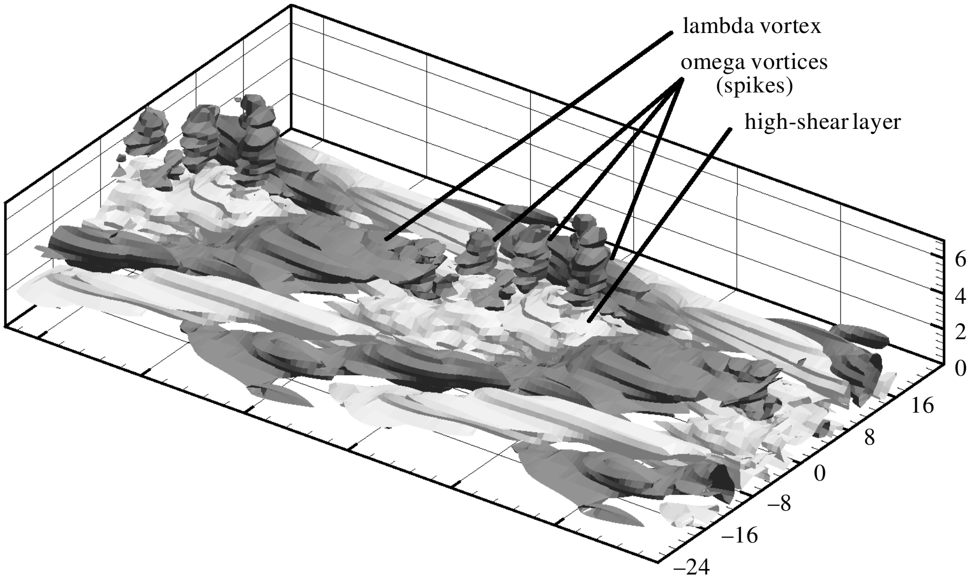


Figure 3. A Λ-vortex at the 3-spike stage of breakdown measured in a wind tunnel by Fernholz & Bake (1998) and made visible by plotting isosurfaces of perturbation velocity u'. ($u'/U_\infty = \pm 6\%$.)

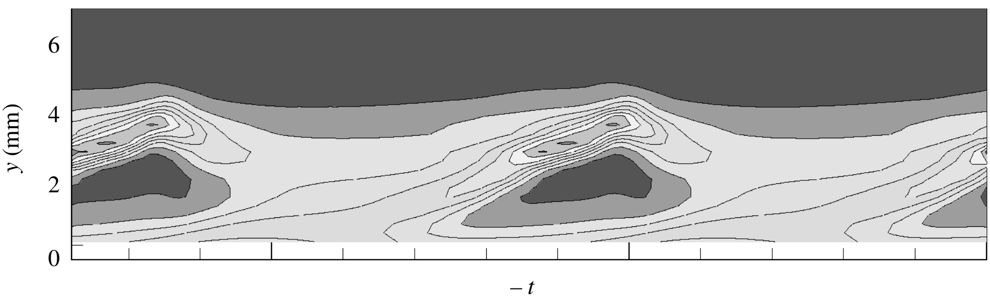


Figure 4. Experimentally measured shear layers, revealed by plotting contours of spanwise vorticity (Fernholz & Bake 1998).

value outside the boundary layer. This rate of increase is termed the profile's shear. A more general description of this rate of change uses the concept of vorticity, which measures the degree of swirl or circulation in the flow about a line. Just above a point in the boundary-layer shear flow the velocity is just greater than that just below (see the profile in figure 1). This may be interpreted as a clockwise swirl or vorticity about the point and, more generally, in a planar flow, about a line parallel to the plate and normal to the direction of flow. When the flow becomes three dimensional in the secondary instability process, these vortex lines become warped and stretched, and motion occurs in a direction parallel to the lines. This causes a flow of swirling fluid towards and along the vortex lines. In just the same way as an ice-skater exploits the conservation of angular momentum to make herself spin faster as she draws her spinning arms in towards herself, this inward motion of fluid causes the swirl to intensify generating vortical structures: the lambda vortices. These have their downstream-

Downloaded from rsta.royalsocietypublishing.org

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

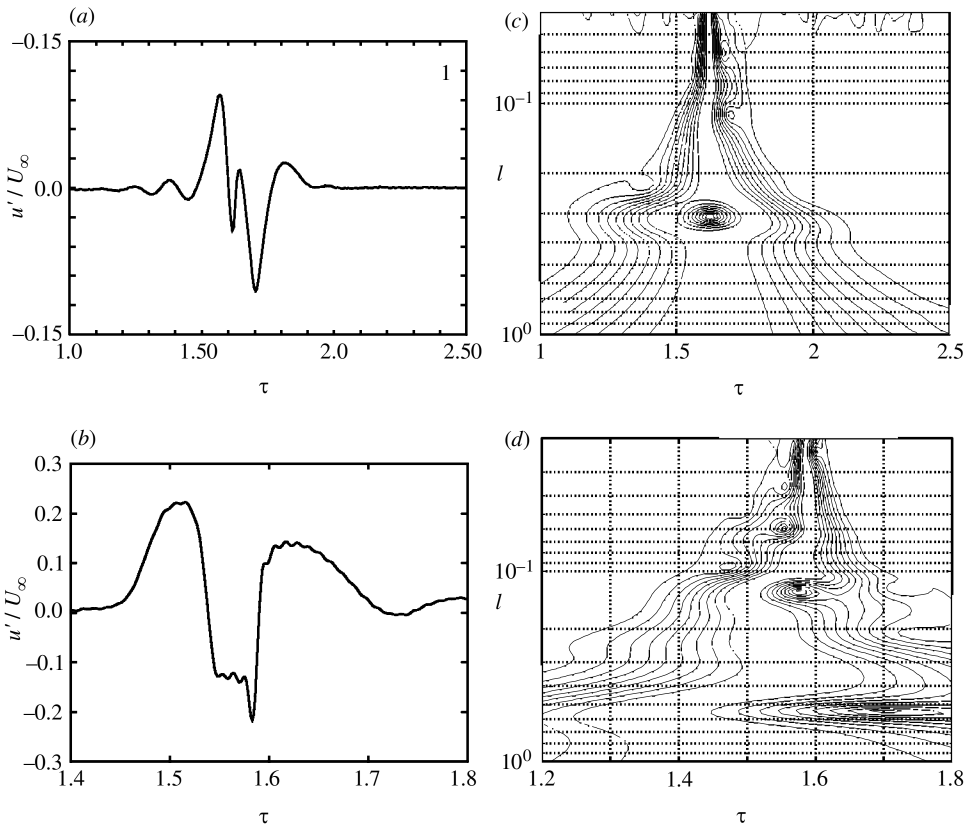


Figure 5. Perturbation velocity (u') traces and their wavelet transforms taken from a transitional wave packet (Breuer *et al*. 1997). (a) A single spike; (b) a secondary instability. Their respective transforms ((c) and (d)) show a cascade of bursts towards higher frequencies. τ and l are a non-dimensionalized time- and length-scale.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

pointing head further from the wall than their trailing legs as they are stretched out in the shear flow. A lambda vortex, measured in a wind tunnel by Sebastian Bake of the Technical University of Berlin, is illustrated in figure 3 (Fernholz & Bake 1998). The circulation about the vortex's legs acts to move slower-moving fluid up from close to the wall, generating delta-wing shaped strong shear layers between the legs. These layers separate this slower-moving fluid from the faster-moving fluid further out in the boundary layer. These shear layers are well illustrated in figure 4.

The experiments of Breuer *et al*. (1997) of Brown University, Providence, Rhode Island, illustrate well the development of the disturbance after the shear layers have been established. They examine the development of a wave packet; a perturbation velocity trace from their experiment is illustrated in figure 5a. There is a rapid drop in perturbation velocity from positive to negative values. This feature is often described as the first spike and corresponds to the passage of a strong shear layer. In the centre of the event is a kink with an associated higher-frequency small-scale structure. This would correspond to the 2-spike stage of the breakdown, but the physics of this

Downloaded from rsta.royalsocietypublishing.org

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

second spike is very different from that of the first, as indicated by its high-frequency content. This second spike is, therefore, often termed the *first* spike. This kink is present in every run of the experiment with only a small variation in its amplitude and position. It is therefore quite repeatable and not associated with turbulence. The three-dimensional structure of the spikes can be seen in figure 3, which corresponds to the 3-spike stage. The spikes in the velocity trace can be associated with the ring-like hairpin or omega (Ω) vortices. These travel downstream, retaining their coherence and interacting only weakly with the rest of the flow (Kachanov 1994).

A trace from further downstream is shown in figure 5*b*. The disturbance has grown in amplitude, but, in addition, there are small-amplitude high-frequency oscillations present. The experiments showed that these oscillations are to be found just upstream of the position of spike generation. They are random in phase and amplitude from realization to realization and their growth leads directly to turbulence. The experiments of Borodulin & Kachanov (1989), described in Kachanov (1994), show that it is possible for these two types of disturbance to coexist. They identified that the spikes had originated further out in the boundary layer than the random oscillations, which could be associated with the shear layer between the legs of the lambda vortex and indeed travel downstream with it.

The structure of these bursts of frequencies higher than the fundamental wave packet may be clarified using the wavelet transform (WT) first applied to boundary-layer transition by Jim Shaikh of Rover Group Ltd, Warwick, UK (Shaikh 1997). The WT decomposes a single time trace into the two-dimensional wavelet plane, with axes of time and frequency by repeated convolution, or comparison of the trace with wavelets of varying scales. A wavelet is, for example, a sinusoidal trace or Fourier mode of a given frequency with a constraining envelope, so that the trace has a given width and decays to zero at both ends. The frequency and width of the wavelet trace vary with the scale of the motion they aim to pick out from the original signal. They are able to isolate short-time-scale high-frequency disturbances in the signal. In contrast, traditional Fourier decomposition into purely periodic waves of a given frequency fails to resolve the localized nature of bursts and measures only a filling of the power spectra as transition proceeds. The wavelet transforms of the velocity traces are shown in figure 5*c*, *d*. In figure 5*d*, the peak at *ca.* $\tau = 1.57$, $l = 0.105$ corresponds to the spikes, while that close to $\tau = 1.55$, $l = 0.06$ is associated with the high-frequency waves.

Sebastian Bake has produced the intriguing figure 6. This plots an isosurface of a measurement of the uncertainty involved in measurements due to the unpredictability of the flow. We can see that this is concentrated in two areas. The first is in the position of the omega vortices. It seems likely that this is associated with slight variations in their formation process. The motion and interaction of vortices can easily lead to chaotic behaviour. The second is associated with the shear layer and may be interpreted as arising from an instability of the shear layer. These two sources of unpredictable behaviour were identified by Sandham & Kleiser (1992) from DNS of channel flow transition.

4. Spiking as wave breaking

The three-dimensional nonlinear development of lower branch TS waves towards breakdown at large R_δ is governed by the so-called triple-deck equations, which

Downloaded from rsta.royalsocietypublishing.org

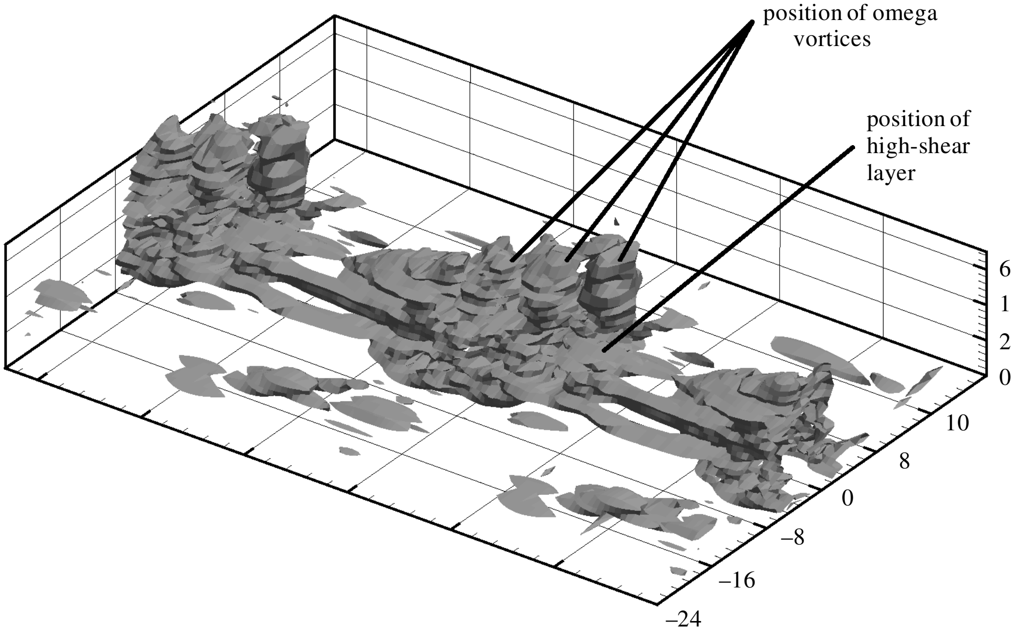


Figure 6. A measurement of the realization-dependent uncertainty of the measurements in figure 3, plotting an isosurface of $u'_{\mathrm{random}}/U_\infty = 3.5\%$ (Fernholz & Bake 1998).

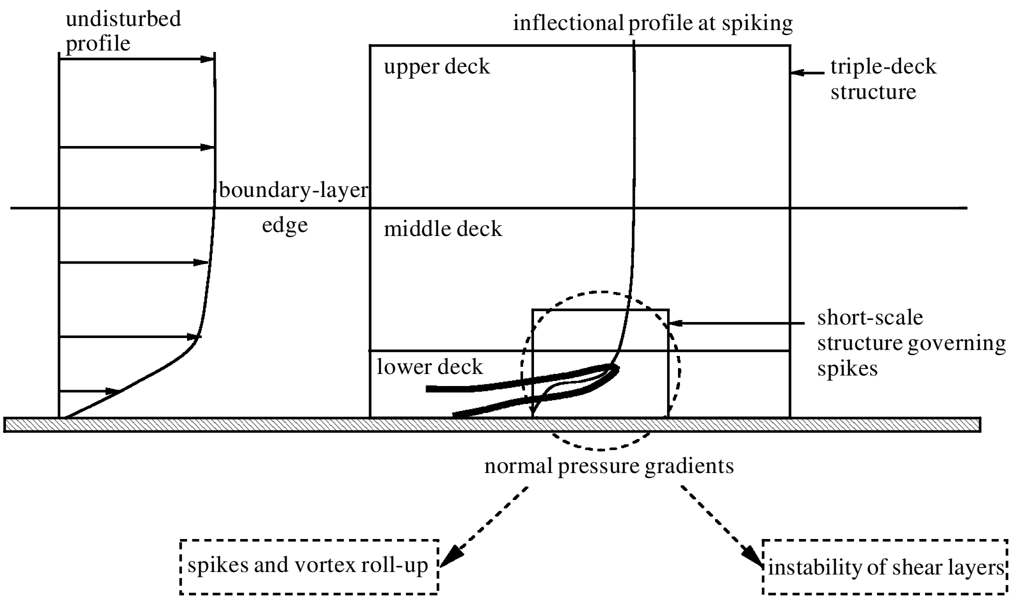


Figure 7. Theoretical aspects of breakdown. The development of the Λ vortex and the shear layer is governed by the triple-deck equations. The finite-time singularity in these is resolved on a shorter scale, allowing normal pressure gradient effects to enter. This gives rise to spikes and vortex roll-up along one transition route and shear-layer instability along a second. In practice, both routes are active simultaneously.

Downloaded from rsta.royalsocietypublishing.org

may be rationally derived from the NS equations (see figure 7). Tollmien–Schlichting waves are long compared with the boundary-layer thickness, but not as long as the scale over which the boundary layer has developed. As a result, viscosity has an influence on their motion over a distance normal to the plate that is less than the full boundary-layer thickness. The background profile here is just a simple constant shear profile. This region close to the plate is the lower deck and corresponds to the merging of the viscous regions at the surface and about the critical layer. The middle deck is a layer whose thickness is that of the boundary layer. The motion here due to the wave is inviscid to first order and corresponds to a vertical heaving motion in response to the flow closer to the wall. Both these regions are relatively long and thin and can support no normal pressure gradients. Instead, the pressure perturbations due to the wave must decay in a region outside the boundary layer of wall-normal extent comparable with the disturbance wavelength. This region is the upper deck. The important constraint on these motions, which fixes the scalings of the lower branch, is that the pressure perturbations in this inviscid upper deck are of the same size as those generated by the viscosity-affected flow close to the wall.

The linearized version of these equations, obtained by neglecting the nonlinear terms, captures the OS solution. A weakly nonlinear version captures the onset of three-dimensional spanwise perturbations (Smith & Walton 1989; Stewart & Smith 1992; Smith & Bowles 1992). The fully nonlinear version is capable of describing the generation of lambda vortices described in § 3. Importantly for the description of the breakdown process, Smith (1988) has shown that the equations have a finite-time singularity, meaning that their solution ceases to exist and cannot be followed after a given time. The singularity has the following characteristics. Firstly, it is associated with a shortening of the streamwise length-scale, with large local values of streamwise pressure gradient and rapid streamwise changes in perturbation velocity. Secondly, it is associated with a sufficiently strong shear layer, in the sense that the instantaneous two-dimensional velocity profile along the centreline of the lambda vortex satisfies a certain mathematical constraint. This constraint relates the velocity profile across the boundary layer to the fluid velocity c at the so-called inflection point, the position of maximum shear. Thirdly, the inflection point coincides with the critical layer of the nonlinear disturbance, which itself travels downstream with speed c.

Smith & Bowles (1992) showed that this condition is closely satisfied by the velocity profiles measured in channel flow transition by Nishioka *et al*. (1979) at the first spike: the rapid drop in perturbation velocity. We can deduce that the onset of the singularity corresponds to this spike. The singularity in the solution of the triple-deck equations implies that they omit some physics vital in the later stages of transition. The shortening length-scale implies that normal pressure gradients must become more important. We must therefore re-examine the NS equations close to this singularity and derive a new set of equations representing the subsequent development of these shorter-scaled, higher-frequency components generated as the singularity is approached. This is described in Li *et al*. (1998). These new equations have solutions reproducing the kinks or higher spikes, the kinks being supported by normal pressure gradients. There are strong analogies with the breaking of an undular bore, a nonlinear free-surface wave seen moving upstream of a steady disturbance introduced into a relatively fast-moving free-surface flow. The initial disturbance to the surface is caused to steepen by nonlinearity and its length-scale shortens. A wave train then develops in the lee of the bore corresponding to the spikes seen in the

Downloaded from rsta.royalsocietypublishing.org

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

transition process. Furthermore, these equations themselves exhibit a singularity as the kinks are generated. This can be removed on a still shorter time-scale active only at this moment, which allows fluid particles to be trapped at the critical layer, forming a region of recirculating flow, i.e. a vortex (Smith *et al.* 2000). This theory, therefore, yields a description of the generation of bursts of high frequencies at the spikes and the concurrent roll-up of the shear layer into vortices, which may be associated with the omega vortices. The equation governing the local pressure in these spikes is similar to the Benjamin–Ono equation, commonly used to model nonlinear dispersive waves and which has soliton solutions that maintain their form as they travel. Kachanov *et al.* (1993) have proposed that spikes are solitons governed by the Benjamin–Ono equation to explain their apparent coherence. This theory, however, modifies the governing equation to include the effects of the critical layer and the presence there of vortices.

The condition on the velocity profile for breakdown to occur has a separate interpretation to that heralding the onset of spiking. It is also the condition for the velocity profile in the lower deck to become unstable to a different instability mechanism, which gives rise to inviscid so-called Rayleigh waves (Bodonyi & Smith 1985; Tutty & Cowley 1986). These waves are possible due to the maximum in the shear in the velocity profile. Here they have wavelengths comparable with the thickness of the lower deck and correspondingly high frequencies, higher than those of both the TS wave and the spikes. The onset of instability has the interesting theoretical feature that, apart from the condition that the waves are relatively long Rayleigh waves and so of low frequency (although still much shorter/higher frequency than TS waves), there is no preferred wavelength for the disturbance. This allows the rational derivation of equations governing a three-dimensional wave packet of weakly nonlinear disturbances (see Savin *et al.* (1998) in a slightly different context and Bowles & Smith (2000)). These waves can be directly associated with the random oscillations of figure 5*b* and the uncertainty or turbulence measured on the shear layer. Indeed, Brown & Smith (1999) have used similar equations to investigate the spreading of a turbulent spot. The theory predicts that the wave packet should primarily move with a speed c, the velocity of the high-shear layer, in agreement with the experiments of Borodulin & Kachanov (1989). The shear layers that arise in cross-flow instability breakdown in a similar way. A study of the stability of velocity profiles generated by DNS by Wintergerste & Kleiser (1998) shows that, in this case, the frequencies present in the wave packet shift to higher values as transition proceeds, although the disturbance retains the same velocity as that of the shear layer. This suggests that, as predicted by the current theory, the disturbance at onset consists of relatively long, low-frequency Rayleigh waves.

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

5. Conclusion

This article has shown how the structure and mechanism of the late stages of transition of boundary layers are being clarified by new experimental and theoretical approaches developed by young researchers. We have seen how the wavelet transform has captured the bursts of high-frequency disturbance that occur. The exact three-dimensional nature of the final breakdown is becoming clear through experimental and computational results. Finally, a theoretical approach that describes transition at high Reynolds numbers as a series of singularities, each triggering the

Downloaded from rsta.royalsocietypublishing.org

onset of new physics, has been presented. In its present form, the theory is mainly only two dimensional, describing only the flow along the centre of the Λ-vortex. Its robustness to three-dimensional extension still needs to be verified. However, its success in incorporating the entrance of normal pressure gradients, vortex roll-up and shear-layer instability show that it offers hope for a theoretical understanding of deep transition and can act as a starting point for exciting future developments.

I thank Jonathan Healey, Sebastian Bake and Kenny Breuer for their help in preparing this paper and supplying figures. Thanks are also due to Jim Shaikh and Frank Smith for many invaluable discussions.

References

Bake, S., Fernholz, H. H. & Kachanov, Y. S. 2000 Resemblance of K- and N-regimes of boundary-layer transition at late stages. *Eur. J. Mech.* B. (In the press.)

Bodonyi, R. J. & Smith, F. T. 1981 The upper branch stability of the Blasius boundary layer including non-parallel flow effects. *Proc. R. Soc. Lond.* A **375**, 65–92.

Bodonyi, R. J. & Smith, F. T. 1985 On the short-scale inviscid instabilities in flow past surface-mounted obstacles and other parallel motions. *Aero J.* (June/July), pp. 205–212.

Borodulin, V. I. & Kachanov, Y. S. 1989 Role of the mechanism of local secondary instability in K-breakdown of boundary layers. *Sov. J. Appl. Phys.* **3**, 70–81.

Bowles, R. I. & Smith, F. T. 2000 Nonlinear instability of the shear layer in end-stage transition. (In preparation.)

Breuer, K. S., Cohen, J. & Horitonidis, J. H. 1997 The late stages of transition induced by a low-amplitude wavepacket in a laminar boundary layer. *J. Fluid Mech.* **340**, 395–411.

Brown, S. N. & Smith, F. T. 1999 Spot concentrations of large-amplitude disturbances in laminar boundary layers. *Q. J. Mech. Appl. Maths* **52**, 269–286.

Fernholz, H. H. & Bake, S. 1998 Experimentelle Untersuchung der Wachstumsmechanisme von 3D-Störungen in einer 2D-Grenzschicht im nichtlinearen Bereich der Transition. Technical Report Fe 43/40-2 DFG, HFI, TU-Berlin.

Goldstein, M. E. 1995 The role of nonlinear critical layers in boundary layer transition. *Phil. Trans. R. Soc. Lond.* A **352**, 425–442

Han, G., Tumin, A. & Wygnanski, I. 2000 Laminar-turbulent transition to a periodic perturbation emanating from the wall. Part II. Late stage of transition. *J. Fluid Mech.* (Submitted.)

Healey, J. J. 1995 On the neutral curve of the flat-plate boundary layer: comparison between experiment, Orr–Sommerfeld theory and asymptotic theory. *J. Fluid Mech.* **288**, 59–73.

Heisenberg, W. 1924 Über Stabilität und Turbulenz van Flussiugkeitsströmen. *Ann. Phys. Lpz.* **74**, 577–627. (English Trans. 1951 On stability and turbulence of fluid flows. *Wash. Tech. Memor. Nat. Adv. Comm. Aero.*, no. 1291.)

Herbert, Th. 1997 Parabolised stability equations. *A. Rev. Fluid Mech.* **29**, 245–283.

Hultgren, L. S. 1987 Higher eigenmodes in the Blasius boundary-layer stability problem. *Phys. Fluids* **30**, 2947–2951.

Kachanov, Y. S. 1994 Physical mechanisms of laminar-boundary-layer transition. *A. Rev. Fluid Mech.* **26**, 411–482.

Kachanov, Y. S., Ryzhov, O. S. & Smith, F. T. 1993 Formation of solitons in transitional boundary layers: theory and experiments. *J. Fluid Mech.* **251**, 273–297.

Klebanoff, P. S., Tidstrom, K. D. & Sargeant, L. M. 1962 The three-dimensional nature of boundary-layer instability. *J. Fluid Mech.* **12**, 1–34.

Klingman, B. G. B., Boiko, A. V., Weshin, K. J. A., Kozlov, V. V. & Alfredsson, P. H. 1993 *Eur. J. Mech.* B **12**, 493–514.

Downloaded from rsta.royalsocietypublishing.org

Lerche, T. 1997 Experimental investigations of nonlinear formation of flow structures in a transitional three-dimensional boundary layer. PhD thesis, University of Göttingen.

Li, L., Walker, J. D. A., Bowles, R. I. & Smith, F. T. 1998 Short-scale breakup in unsteady interactive layers: local development of normal pressure gradients and vortex wind-up. *J. Fluid Mech.* **374**, 335–378.

Lighthill, M. J. 1963 Introduction to boundary layer theory. In *Laminar boundary layers* (ed. L. Rosenthal), pp. 88–94. Oxford University Press.

Metcalfe, R. W., Baattistoni, F., Orzo, S. & Ekeroot, J. 1991 Evolution of boundary layer flow over a compliant wall during transition to turbulence. In *Royal Aeronautical Society Conf. on Boundary Layer Transition and Control, Cambridge, 8–12 April*.

Nishioka, M., Asai, M. & Iida, S. 1979 An experimental investigation of the secondary instability. In *Laminar-turbulent transition. IUTAM Mtg, Stuttgart.*

Orr, W. M. F. 1907 The stability or instability of the steady motions of a perfect liquid and of a viscous liquid. *Proc. R. Irish Acad.* A **27**, 9–68, 69–138.

Reid, W. H. 1965 In *Basic developments in fluid dynamics* (ed. M. Holt), vol. 1, pp. 249–307. Academic.

Ross, J. A., Barnes, F. H., Barnes, J. G. & Ross, M. A. S. 1970 The flat plate boundary layer. Part 3. Comparison of theory with experiment. *J. Fluid Mech.* **43**, 819–838.

Sandham, N. D. & Kleiser, L. 1992 The late stages of transition to turbulence in channel flow. *J. Fluid Mech.* **245**, 319–348.

Savin, D. J., Smith, F. T. & Allen, T. 1998 Transition of free disturbances in inflectional flow, over an isolated surface roughness. *Proc. R. Soc. Lond.* A **455**, 491–541.

Schlichting, H. 1933 Zur Entstewhung der Turbulenz bei der Plattenströmung. *Nachr. Ges. Wiss. Göttmaen, Math. Phys. K. L.*, pp. 181–208.

Schubauer, G. B. & Skramstad, H. K. 1943 Laminar-boundary-layer oscillations on a flat plate. NACA report, no. 909.

Shaikh, F. N. 1997 Investigation of transition to turbulence using white-noise excitation and local analysis techniques. *J. Fluid Mech.* **348**, 29–83.

Smith, F. T. 1979*a* On the nonparallel stability of the Blasius boundary layer. *Proc. R. Soc. Lond.* A **366**, 91–109.

Smith, F. T. 1979*b* Nonlinear stability of boundary layers for disturbances of various sizes. *Proc. R. Soc. Lond.* A **368**, 573–589.

Smith F. T. 1988 Finite time break-up can occur in any unsteady interacting boundary-layer. *Mathematika* **35**, 256–273.

Smith, F. T. & Bowles, R. I. 1992 Transition theory and experimental comparisons on (a) amplification into streets and (b) a strongly nonlinear break-up criterion. *Proc. R. Soc. Lond.* A **439**, 163–175.

Smith, F. T. & Walton, A. G. 1989 Nonlinear interaction of near-planar TS waves and longitudinal vortices in boundary-layer transition. *Mathematika* **36**, 262–289.

Smith, F. T., Bowles, R. I. & Walker, J. D. A. 2000 Wind-up of a spanwise vortex in deepening transition and stall. *Theoret. Comp. Fluid Dyn.* (Submitted.)

Sommerfeld, A. 1908 Ein Beitrag zur hydrodynamischen Erklaerung der turbulenten Fluessigkeitsbewegungen. In *Proc. 4th Int. Congr. Mathematicians, Rome*, vol. III, pp. 116–124.

Stewart, P. A. & Smith, F. T. 1992 Three-dimensional nonlinear blow-up from a nearly planar initial disturbance in boundary-layer transition: theory and experimental comparisons. *J. Fluid Mech.* **244**, 649–676.

Tollmien, W. 1929 Über die Entstehung der Turbulenz. *Nachr. Ges. Wiss. Göttmaen, Math. Phys. K. L.*, pp. 21–44. (English Trans. 1931 The production of turbulence. *Tech. Memor. Nat. Adv. Comm. Aero. Wash.*, no. 609.)

Tutty, O. R. & Cowley, S. J. 1986 On the stability and the numerical solution of the unsteady interactive boundary-layer equation. *J. Fluid Mech.* **168**, 431–456.

Downloaded from rsta.royalsocietypublishing.org

Walker, J. D. A. 1990*a* Wall-layer eruptions in turbulent flows. In *Proc. 2nd IUTAM Symp. on Structure of Turbulence and Drag Reduction, Zurich, Switzerland* (ed. A. Gyr). Springer.

Walker, J. D. A. 1990*b* Models based on dynamical features of the wall layer. In *Proc. XI US Natl Congr. of Applied Mathematics, University of Arizona, Tucson, AZ, 21–25 May; Appl. Mech. Rev.* **43**, S232–S239.

Wintergerste, T. & Kleiser, L. 1995 Direct numerical simulation of transition in a three-dimensional boundary layer. In *Proc. Symp. on Boundary-Layer Transition Prediction in Aeronautics, Amsterdam, December*. Springer.

Wintergerste, T. & Kleiser, L. 1998 Secondary instability analysis of nonlinear cross-flow vortices. *Euromech 380 Conf. Laminar-Turbulent Mechanisms and Prediction, Göttingen, 14–17 September.*

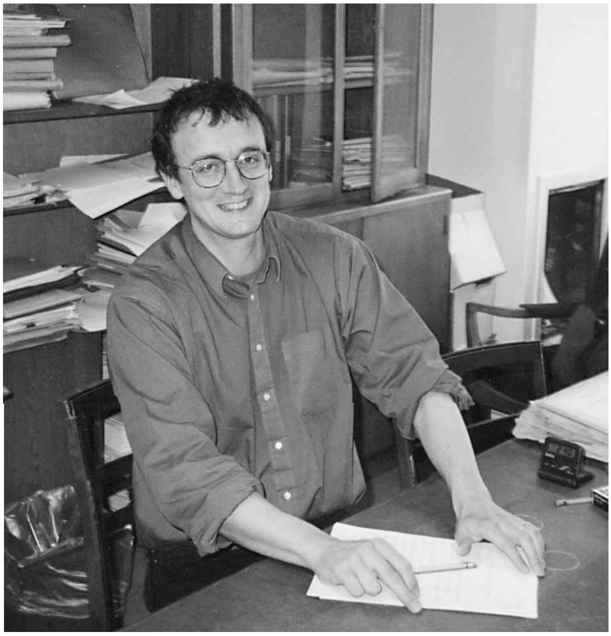
Downloaded from rsta.royalsocietypublishing.org

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

AUTHOR PROFILE

R. I. Bowles

Robert Bowles obtained a degree in mathematics from Bristol University in 1985 and proceeded to study at Cambridge before commencing a PhD under the supervision of Professor F. T. Smith at University College London in 1986. He was appointed Lecturer in the Department of Mathematics there, becoming a senior lecturer in 1995. His research interests include the fluid mechanics of liquid-layer flows, boundary-layer transition and two-fluid boundary layers. He also has a strong interest in teaching, especially of mathematics to first-year undergraduates. He is 35, married with two children, and lives in Hertfordshire, where much of his spare time is taken up with play and keeping the garden under control.



MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF